ASSIGNMENT 2
DISCRETE STRUCTURE
(SECI1013)

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Q1 Relation

1. $A = \{2, 3, 4, 5, 6, 7, 8\}$

$x - y = 3n$

$n = \dfrac{x-y}{3} \in Z$

$R = \{(2,2), (2,5), (2,8), (3,3), (3,6), (4,4), \cancel{(4,6)}, (4,7), (5,2), (5,5), (5,8), (6,3$
$(6,6), (7,4), (7,7), (8,2), (8,5), (8,8)\}$

2. $A = \{1, 2, 3\}$ $B = \{9, 8, 7\}$

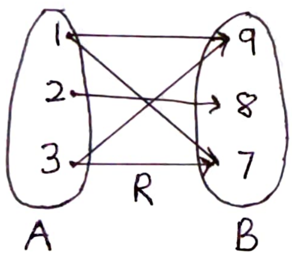
$aRb \leftrightarrow a+b$ is an even number

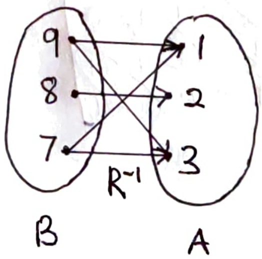
a. $R = \{(1,7), (1,9), (2,8), (3,7), (3,9)\}$

$R^{-1} = bRa$

$= \{(7,1), (9,1), (8,2), (7,3), (9,3)\}$

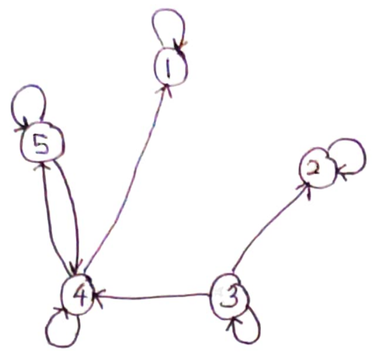
b.





c. R^{-1} ~~represent~~ is the inverse of relation R which make R^{-1} is bRa. ~~This~~ ~~Represent that~~ ~~sum of b + a is an even~~ This mean A and B swap places.

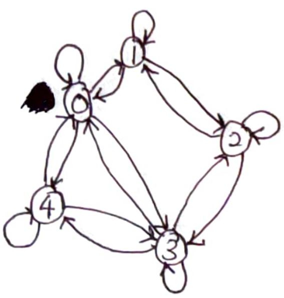
3. $A = \{1, 2, 3, 4, 5\}$



	1	2	3	4	5
In - degree	2	2	1	3	2
Out - degree	1	1	3	3	2

4. $A = \{0, 1, 2, 3, 4\}$

$R = \{(0,0), (0,1), (0,3), (0,4), (1,0), (1,1), (1,2), (2,1), (2,2), (2,3), (3,0), (3,2), (3,3), (3,4), (4,0), (4,3), (4,4)\}$



$$\begin{array}{c c} & \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow \text{not transitive}$$

$\therefore$ R is reflexive and symmetric

5. $A = \{1, 2, 3 \ldots 13, 14\}$ $R = \{(1,3), (2,6), (3,9), (4,12)\}$

$R = \{(x,y) : 3x - y = 0\}$

a. ~~Reflexive~~ This relation is not reflexive. Because ~~(x, y)~~ x and y cannot be same number ~~except~~ when put into equation ~~except (0,0)~~. For example let ~~assume~~ assume $x = 2, y = 2$ ∴ $3(2) - 2 \neq 0$.

b. This relation is not symmetric. Because ~~∀x,y ∈ A There are None~~ none from ~~set A~~ relation have symmetric. For example, $(1,3) \in R$ but $(3,1) \notin R$.

c. This relation is not transitive. Because ~~None £~~ none from relation ~~have~~ transitive. For example, $(1,3)$ and $(3,9) \in R$ but $(1,9) \notin R$

6. $R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

a. RS

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$0+0+0+0=0$
$0+0+1+0=1$
$0+0+1+1=1$
$0+0+0+1=1$
$1+0+0+0=1$
$0+1+0+0=1$
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$0+0+0+0=0$
$0+0+0+1=1$
$0+0+0+1=1$

b. SR

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$0+0+0+0=0$
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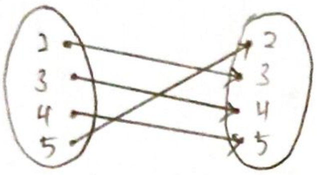
$0+0+0+0=0$
$0+0+0+0=0$
$0+0+1+0=1$
$0+0+1+1=1$

Q2. Function

7. Relation are natural to associate objects of various set while function is a special type of relations with certain characteristic.

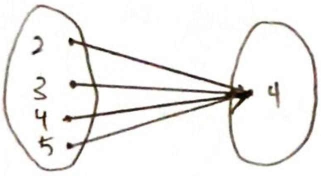
8. $A = \{2, 3, 4, 5\}$

i) ~~The ta relation~~



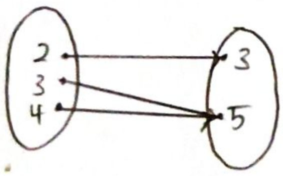
This relation is a function because ~~there is only one arrow from every element~~ all the element of domain are ~~all~~ in set A

ii)



This relation is a function because all the element of domain are ~~all~~ in set A ~~but~~ even all the elements are assigned to the same value

iii)



This relation is not a function because the domain is missing one element from set A

iv) ~~The first relation is a~~

This relation is not a relation because ~~the~~ there are two element that are assigned ~~two~~ twice to ~~but~~ the different value.

9. $R = \{(1,6),(2,7),(3,8),(4,9)(5,10)\}$

Domain $= \{1,2,3,4,5\}$
Range $= \{6,7,8,9,10\}$

10. v) let $f(x_1) = f(x_2)$

$$1 - 2x_1 = 1 - 2x_2$$
$$2x_1 = 2x_2$$
$$x_1 = x_2$$

$\therefore$ this function is ~~one to~~ one-one function

let $f(x) = y$
$y = 1 - 2x$

if x is real number, then y can be also ~~are~~ a real number
Therefore, it is an onto function

$\therefore$ This function is a bijective function

vi) let $f(x_1) = f(x_2)$

$$f(-2) = f(2)$$

$\therefore$ this function is not one-one function

let $y = f(x)$

$y = 5x^2 - 1$

~~y will~~

$\therefore$ x will always be a positive number not matter what the value is. The y will have the same value with a different x
Therefore, it is not an onto function

vii) let $f(x_1) = f(x_2)$

$$f(-2) = f(2)$$

$\therefore$ This is not one-one function

let $f(x) = y$
$y = x^4$

$\therefore$ y will ~~always~~ always be a positive value ~~not~~.
Therefore, it is not an onto function

viii) let $f(x_1) = f(x_2)$

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$x_1x_2 - 2x_2 - 3x_1 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$$

$$-2x_2 - 3x_1 = -3x_2 - 2x_1$$
$$x_1 = x_2$$

$\therefore$ This is not one-one function

let $f(x) = y$

$$y = \frac{x - 2}{x - 3}$$

y can be a real number, when x is also a real number except $x = 3$ because of denominator.
~~There~~ Therefore, it is an onto function

$\therefore$ This function is bijective

11. ix) $f(g(x)) = f(x^2-1)$

$= 3(x^2-1)-1$

$= 3x^2-3-1$

$= 3x^2-4$

$f(g(0)) = 3(0)^2-4 = 0$

$f(g(1)) = 3(1)^2-4 = -1$

$f(g(2)) = 3(2)^2-4 = 8$

$f(g(3)) = 3(3)^2-4 = 23$

x) $f(g(x)) = f(5x-6)$

$= (5x-6)^2$

$= (5x-6)(5x-6)$

$= 25x^2-60x+36$

$f(g(0)) = 25(0)^2-60(0)+36 = 36$

$f(g(1)) = 25(1)^2-60(1)+36 = 1$

$f(g(2)) = 25(2)^2-60(2)+36 = 16$

$f(g(3)) = 25(3)^2-60(3)+36 = 81$

xi) $f(g(x)) = f(x^3+1)$

$= (x^3+1)-1$

$= x^3$

$f(g(0)) = 0^3 = 0$

$f(g(1)) = 1^3 = 1$

$f(g(2)) = 2^3 = 8$

$f(g(3)) = 3^3 = 27$

Q3 Recurrence Relation

12. (xii) $a_0 = 1$

$a_1 = 6$

$a_2 = 6a_1 - 9a_0$

$= 6(6) - 9(1) = 27$

$a_3 = 6(27) - 9(6) = 108$

$a_4 = 6(108) - 9(27) = 405$

1, 6, 27, 108, 405, 1458, 5103, 17496, ...

(xiii) $a_0 = 2$

$a_1 = 5$

$a_2 = 15$

$a_3 = 6a_2 - 11a_1 + 6a_0$

$= 6(15) - 11(5) + 6(2) = 47$

$a_4 = 6(47) - 11(15) + 6(5) = 147$

$a_5 = 6(147) - 11(47) + 6(15) = 455$

2, 5, 15, 47, 147, 455, 1395, 4247, 12867, ...

(XIV) $a_0 = 1$

$a_1 = -2$

$a_2 = -1$

$a_3 = -3a_2 - 3a_1 + a_0$

$= -3(-1) - 3(-2) + (1) = 10$

$a_4 = -3(10) - 3(-1) + (-2) = -29$

$a_5 = -3(-29) - 3(10) + (-1) = 56$

$1, -2, -1, 10, -29, 56, -71, 16, 221, -782, \ldots$

13 i) $a_2 = 5a_1 - 3$

$a_3 = 5a_2 - 3$

$= 5(5a_1 - 3) - 3$

$= 25a_1 - 15 - 3$

$a_3 = 25a_1 - 18$

$a_4 = 5a_3 - 3$

$= 5(25a_1 - 18) - 3$

$= 125a_1 - 90 - 3$

$a_4 = 125a_1 - 93$

$\therefore a_4 = 125k - 93$

ii) $a_4 = 125k - 93$

$7 = 125k - 93$

$125k = 100$

$k = \frac{4}{5}$ *